A Tableau System for Right Propositional Neighborhood Logic over Finite Linear Orders: an Implementation

TABLEAUX-2013

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- Nevertheless, some experimental work, such as this one, are not free of unexpected problems and difficulties.
- Moreover, this implementation, although particularly simple, is the only one of its kind.

Time and logics

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It is well-known that interval-based logics are much more difficult to deal with.

What is an interval?

Definition

Given a linear order $\mathbb{D} = \langle D, < \rangle$:

- an interval in $\mathbb{D}$ is a pair $[d_0, d_1]$ such that $d_0 < d_1$;
- $\mathbb{I}(\mathbb{D})$ is the set of all (strict) intervals on $\mathbb{D}$;
- $\langle \mathbb{D}, \mathbb{I}(\mathbb{D}) \rangle$ is an interval structure.

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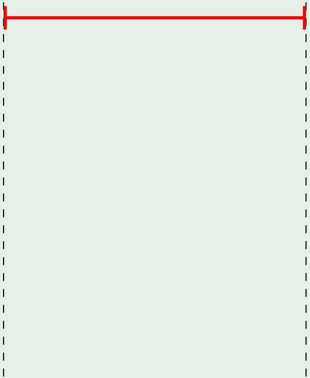
- We consider intervals as pairs of time points.
- A point $d \in D$ belongs to $[d_0, d_1]$ if $d_0 \leq d \leq d_1$.
- Sometimes, the non-strict semantics, where intervals may have coincident endpoints, is considered; nowadays, the common choice is to exclude this possibility, treating points as a different sort, and giving rise to more complex point-interval logics.

Allen's binary relations

There are 13 different binary relations between intervals:



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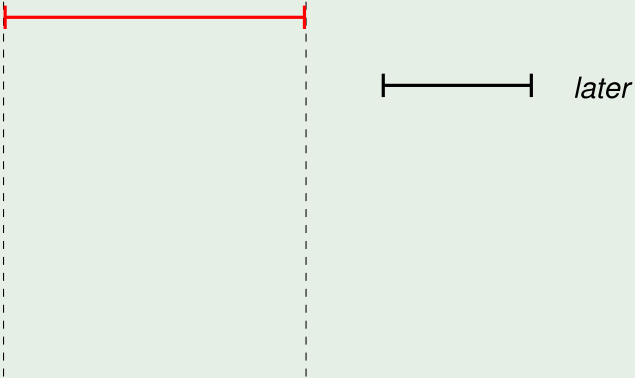

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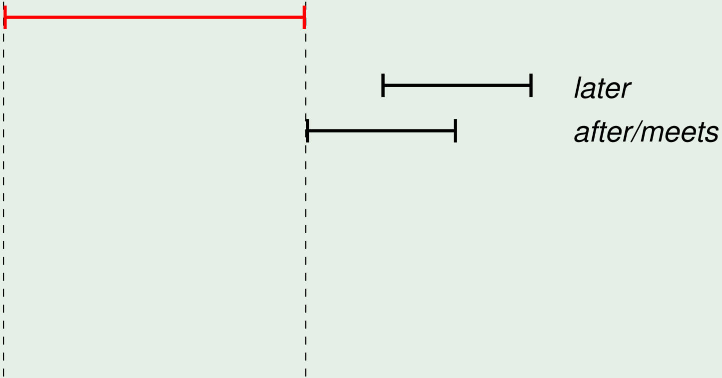

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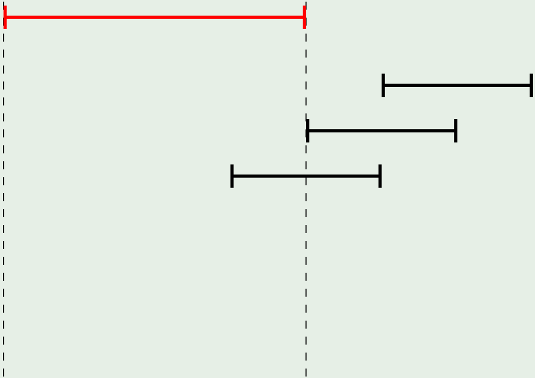

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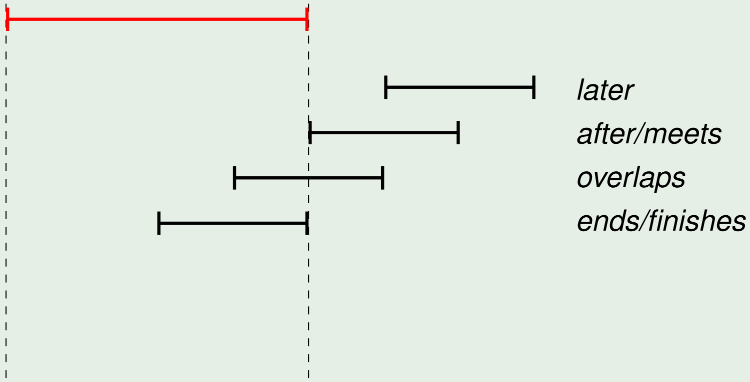

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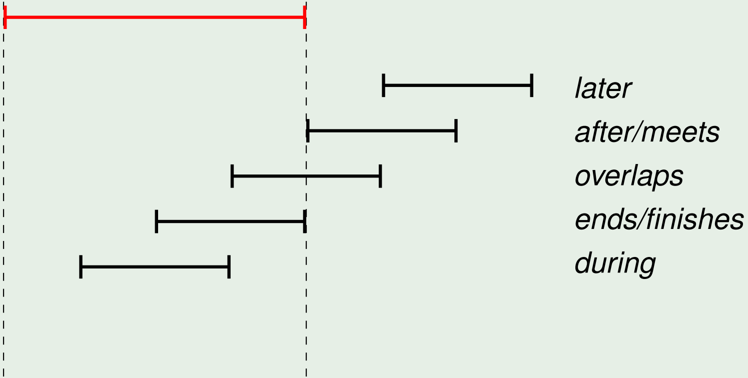

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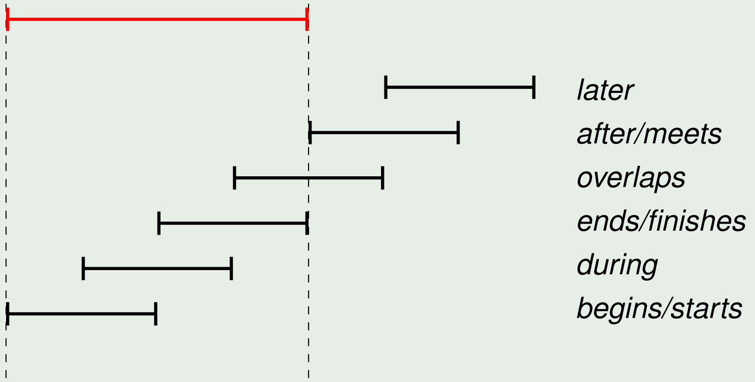

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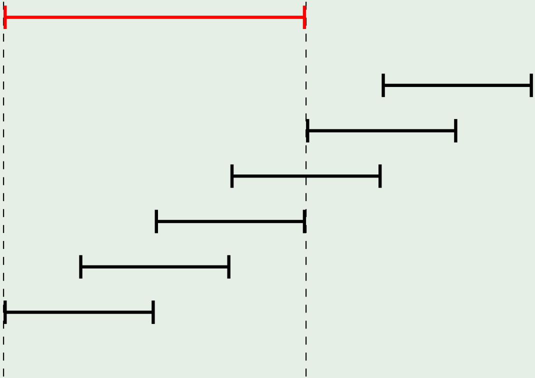

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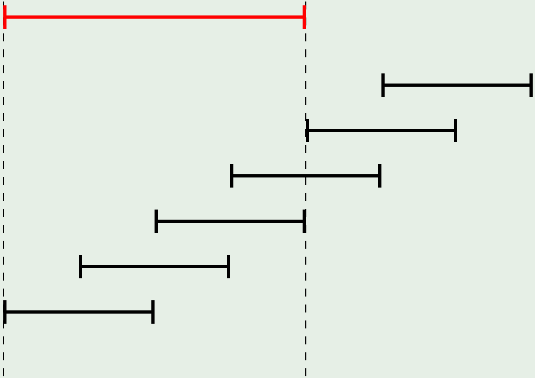

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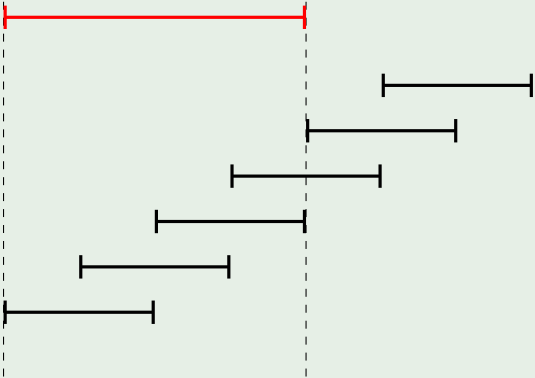

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Between points we have only three binary ordering relations!

Halpern-Shoham's modal logic of interval relations

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Syntax of Halpern-Shoham's logic, hereafter called HS :

$$\phi ::= p \mid \neg\phi \mid \phi \wedge \psi \mid \langle B\rangle\phi \mid \langle E\rangle\phi \mid \langle \overline{B}\rangle\phi \mid \langle \overline{E}\rangle\phi \mid \langle A\rangle\phi \mid \langle \overline{A}\rangle\phi.$$

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Interval model:

$$\mathbf{M} = \langle \mathbb{I}(\mathbb{D}), V\rangle,$$

where $V : \mathcal{AP} \mapsto 2^{\mathbb{I}(\mathbb{D})}$.

Formal semantics of HS

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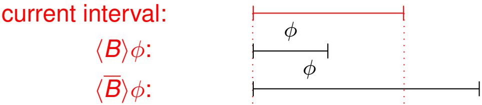

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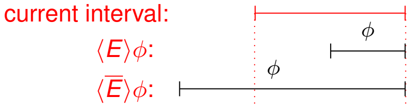

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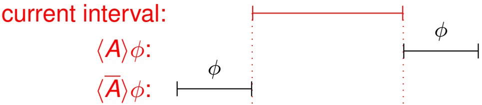

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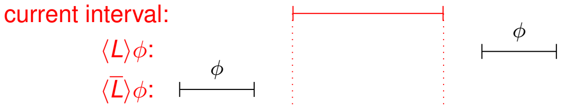

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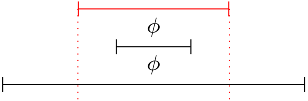

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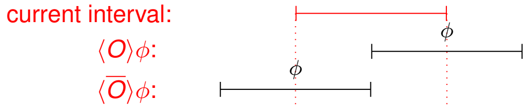

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We focus here on the class of all finite linearly ordered sets.

Classifying HS fragments on finite linear orders - 1

Complexity class:

1: Non primitive recursive

2: EXPSPACE-complete

3: NEXPTIME-complete

4: NP-complete

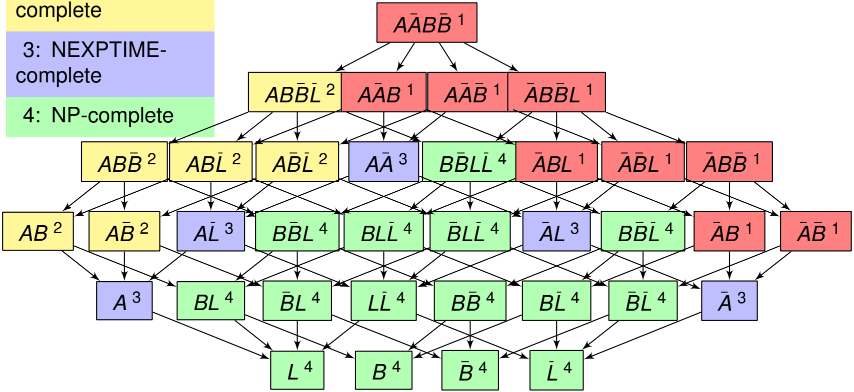

Figure : Hasse diagram of all and only decidable fragments of HS over finite linear orders.

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In this work, we use the results we have that concern the fragment A in the finite case, and we put them at work to build a usable satisfiability checker.

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In this work we adapted the 2003 result (a classical tableaux) with the closing condition from the 2007 result, to obtain a usable, optimizable, and relatively easy to implement method.

The theoretical basis for A-formulas satisfiability checking - 2

The main theorem

Let φ be a A-formula. Then, φ is finitely satisfiable if and only if it is satisfiable on a model whose cardinality is strictly less than $2^m \cdot m + 1$, where m is the number of diamonds and boxes in φ.

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Clearly, this result shows that A is NEXPTIME; in 2007 we also proved that it is NEPTIME-complete.

A-rules - 1

Given a A-formula φ, we want to establish if there exists a finite model $\mathbf{M}$ and an interval $[d_i, d_j]$ in it such that $\mathbf{M}, [d_i, d_j] \Vdash \varphi$.

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A run of the tableau determines if a given formula φ is satisfiable over the interval $[d_0, d_1]$; thus, a *node* is a *labelled formula* of the type $\psi, [d_i, d_j]$.

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where for each $j \leq h \leq N$, d_h is a point in D and d'_h is a new point added to D and placed immediately after d_h and immediately before d_{h+1} (when $h < N$).

So, every application of the classical $\vee$-rule a new branch is created, on which we keep trace of the current domain D.

A branch is declared *closed* in one of two situations: either we find a contradiction (i.e., both p and $\neg p$ are labeled with $[d_i, d_j]$ for some proposition p), or the cardinality of the domain D associated with the branch is too big (termination condition). Otherwise the branch is *open*, and rules are applied only to open branches.

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Branch-expansion is performed top-down: the first active formula is found and its (only) applicable rule is applied to it. If no more active formulas are found on a open branch, then the system terminated with success (satisfiable formula); if all branches have been closed, then the result is negative (unsatisfiable formula).

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A *branch* B is a list of nodes, enriched with two integer variables N and A representing, respectively, the cardinality of the domain D_B and the number of active nodes.

Search Procedure

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The search procedure first selects the branch with highest priority; then, checks if it meets any closure condition, in which case the branch is closed and deleted, and the procedure re-starts; then, it selects the closest-to-the-root active node, and if there are none, the formula is declared satisfiable, being the current branch a model for it; finally, it applies the expansion rule to the selected node.

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If as a result of the search procedure all branches are deleted, the formula is declared unsatisfiable.

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The policy is decided before starting the search procedure and cannot be changed during a run.

Experimental Results - 1

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Randomized: a series of 36 completely random problems, generated in a suitable clausal form, to simulate the behaviour in real cases.

Experimental Results - 2

COMBINATORICS

n	Policy (sec)					Outcome
n	FIFO	SDF	LDF	SAN	GAN	(size)
1	0.004	0.004	0.004	0.004	0.004	4
2	0.004	0.008	0.004	0.004	0.008	5
3	0.008	0.15	0.03	0.008	0.03	6
4	0.01	–	30.07	0.01	30.29	7
5	0.012	–	–	0.012	–	8
6	0.02	–	–	0.03	–	9
7	0.07	–	–	0.07	–	10
8	0.15	–	–	0.16	–	11
9	0.3	–	–	0.32	–	12
10	0.56	–	–	0.59	–	13
11	0.99	–	–	1.06	–	14

n	Policy (sec)					Outcome
n	FIFO	SDF	LDF	SAN	GAN	(size)
12	1.67	–	–	1.79	–	15
13	2.73	–	–	2.94	–	16
14	4.25	–	–	4.55	–	17
15	6.56	–	–	7.08	–	18
16	9.77	–	–	10.82	–	19
17	14.42	–	–	15.40	–	20
18	20.79	–	–	22.20	–	21
19	29.28	–	–	32.11	–	22
20	40.91	–	–	44.09	–	23
21	–	–	–	–	–	–
22	–	–	–	–	–	–

Experimental Results - 3

RANDOMIZED

n	Policy (sec) FIFO	SDF	LDF	SAN	GAN	Outcome (size)
1	0.004	0.004	0.004	0.004	0.004	4
2	0.004	0.004	0.004	0.004	0.004	4
3	0.004	0.004	0.004	0.004	0.004	4
4	0.004	0.004	0.004	0.004	0.004	4
5	0.004	0.004	0.004	0.004	0.004	4
6	0.004	0.004	0.004	0.004	0.004	4
7	0.07	0.23	0.004	0.18	0.004	3 / 4
8	0.004	0.004	0.004	0.004	0.004	4
9	0.004	0.004	0.004	0.004	0.004	4
10	0.004	0.004	0.004	0.004	0.004	4
11	0.004	0.004	0.004	0.004	0.004	4
12	0.004	0.004	0.004	0.004	0.004	4
13	0.01	0.04	0.004	0.02	0.004	4
14	0.004	0.004	0.004	0.004	0.004	4
15	0.004	0.004	0.004	0.004	0.004	4
16	0.004	1.37	0.004	0.01	0.004	4
17	0.004	0.004	0.004	0.004	0.004	4
18	0.004	0.004	0.004	0.004	0.004	3

n	Policy (sec) FIFO	SDF	LDF	SAN	GAN	Outcome (size)
19	1.66	45.43	0.68	1.91	0.02	3 / 4
20	0.02	0.004	0.03	0.03	0.004	2 / 4
21	0.004	0.004	0.004	0.004	0.004	4
22	0.74	14.08	0.004	1.04	0.004	4
23	0.004	0.004	0.004	0.004	0.004	4
24	0.004	0.004	0.004	0.004	0.004	4
25	–	–	–	–	–	–
26	0.004	0.004	0.004	0.004	0.004	4
27	0.004	–	0.004	0.01	–	3 / 4
28	0.004	0.004	0.004	0.004	0.004	4
29	0.004	–	0.004	0.004	0.004	4
30	0.14	0.08	0.04	0.19	0.01	2 / 4
31	0.004	0.004	0.004	0.004	0.004	unsat
32	0.25	–	0.02	0.31	0.004	2 / 4
33	0.004	0.004	0.004	0.004	0.004	4
34	–	–	0.02	0.004	0.02	2 / 4
35	0.004	–	0.004	–	0.004	2 / 4
36	–	–	–	–	1.2	3

Conclusions

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Unfortunately, even for simple decidable cases, not much effort has been devoted to the implementation and realization of the theoretical procedures. We know that there are expressive enough and relatively computationally simple interval-based languages that can be used in many practical applications, but the gap with the reality of the implementations is still to be filled.

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This work represents a first step in this direction. On the web page *http://www.di.unisa.it/dottorandi/dario.dellamonica/tableaux/* it is possible to find the system available for testing. We plan to expand and improved it to deal with more expressive languages and with different classes of linear orders. Moreover, we also plan to solve a new problem that we found during our experiments: there is no literature available on the task of generating a proper benchmark in the interval case.