Dealing with Symmetries in Modal Tableaux

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Introduction

A symmetry is a permutation of the variables (literals) of a problem that preserves its structure and its set of solutions.

For instance:

$$\varphi = (\neg p \vee r) \wedge (q \vee r) \wedge \Box(\neg p \vee q)$$

has symmetry:

$$\rho = (\neg p \; q)(\neg q \; p)$$

We may improve the performance of theorem proving if:

- symmetry detection is cheap
- this information pays off in terms of performance

We present a tableau optimization called "symmetry blocking".

Syntax

Modal Conjunctive Normal Form:

- Clausal representation of modal formulas:

$$(\neg p \vee r) \wedge (q \vee r) \wedge (r \vee \Box(\neg p \vee q)))$$
$$\rightarrow \quad \{\{\neg p, r\}, \{q, r\}, \{r, \Box\{\neg p, q\}\}\}$$

- Disregard order and multiplicity: formulas as set of sets.

Symmetry:

- Permutations of literals, $\rho : \text{PLIT} \mapsto \text{PLIT}$
- ρ is a **symmetry of** φ if $\rho(\varphi) = \varphi$.

Semantics

Models:

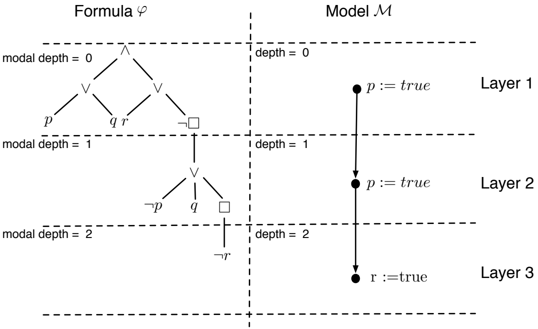
- Kripke model: $\mathcal{M} = \langle W, R, V \rangle$
  - W is the domain
  - $R \subseteq W \times W$
  - $V : W \mapsto \mathcal{P}(\text{PROP})$
  - Pointed Models: $\mathcal{M} = \langle w, W, R, V \rangle$, $w \in W$

Satisfaction Relation:

$\mathcal{M} \models \varphi$	iff	$\mathcal{M} \models C$ for *all* clauses $C \in \varphi$
$\mathcal{M} \models C$	iff	$\mathcal{M} \models l$ for *some* literal $l \in C$
$\mathcal{M} \models p$	iff	$p \in V(w)$ for $p \in \text{PROP}$
$\mathcal{M} \models \Box C$	iff	$\langle w', W, R, V \rangle \models C$ for all w' s.t. wRw'

Permutation Sequences

- In modal logics that have the *tree model property*, a notion of *layer* is induced:

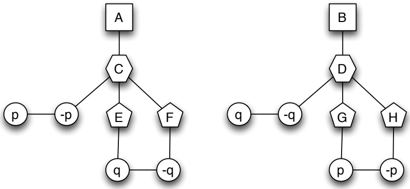


- We can consider a different permutation at each layer.
- Permutation Sequence: $\bar{\rho} = \langle \rho_1, \ldots, \rho_n \rangle$
- Enables to find more symmetries.

Symmetry Detection

- Create a graph from the formula such that its automorphism group is isomorphic to the symmetry group of the formula.
- Pass it to graph automorphism tools (eg Saucy, Bliss).
- The reduction enables the detection of layered symmetries.
- Node coloring avoids spurious symmetries.

$$\varphi = \neg\Box(\neg p \vee \Box q \vee \Box\neg q) \wedge \neg\Box(\neg q \vee \Box p \vee \Box\neg p)$$



$$
\begin{aligned}
A &= \neg\Box(\neg p \vee \Box q \vee \Box\neg q)\\
B &= \neg\Box(\neg q \vee \Box p \vee \Box\neg p)\\
C &= \neg p \vee \Box q \vee \Box\neg q\\
D &= \neg q \vee \Box p \vee \Box\neg p\\
E &= \Box q\\
F &= \Box\neg q\\
G &= \Box p\\
H &= \Box\neg p
\end{aligned}
$$

Group Generators:

$$
\begin{aligned}
\bar\rho_1 &= \langle \rho_{Id}, \rho_{Id}, (p\ \neg p)\rangle\\
\bar\rho_2 &= \langle \rho_{Id}, \rho_{Id}, (q\ \neg q)\rangle\\
\bar\rho_3 &= \langle \rho_{Id}, (p\ q)(\neg p\ \neg q), (p\ q)(\neg p\ \neg q)\rangle
\end{aligned}
$$

Symmetry Detection: Experimental Evaluation

Testbed:

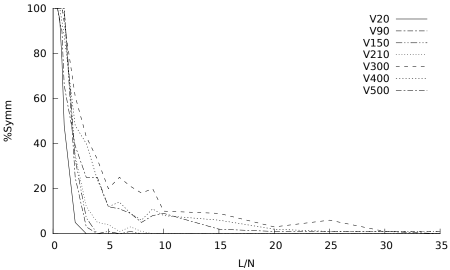
- 378 LWB_K+ 756 QBFLib + 19000 random.

Conclusions:

- Symmetries do arise in modal formulas.
- As expected: encoding of the formula drives the existence of symmetries (LWB_K).
- % of symmetries in random instances highly depends on L/N.

	#Inst	#Sym	T
LWB_K	378	208	10.2
QBFLib	756	746	16656

Class	#Inst	#Sym	AvGen
k_branch	42	42	12
k_d4	42	0	0
k_dum	42	0	0
k_grz	42	42	4
k_lin	42	1	1
k_path	42	42	35
k_ph	42	39	1
k_poly	42	42	18
k_t4p	42	0	0



BML Prefixed Tableaux Calculus

$$\frac{\sigma{:}\varphi}{\sigma{:}C_i}\,(\wedge) \quad \text{for all } C_i \in \varphi \qquad\qquad \frac{\sigma{:}C}{\sigma{:}l_1 \mid \ldots \mid \sigma{:}l_n}\,(\vee) \quad \text{for all } l_i \in C$$

$$\frac{\sigma{:}\neg\Box C}{\sigma R \sigma',\ \sigma'{:}{\sim}C}\,(\Diamond)^1 \qquad\qquad \frac{\sigma{:}\Box C, \sigma R \sigma'}{\sigma' : C}\,(\Box)$$

[1] $\sim C$: CNF of the negation of C. The prefix σ' is new in the tableau.

- A branch is *closed* if contains both $\sigma{:}p$ and $\sigma{:}\neg p$, otherwise *open*.
- A branch is *saturated* if no rule can be further applied.

Symmetry Blocking

We write $\Gamma(\sigma) = \{\psi \mid \sigma{:}\Box\psi \in \Theta\}$ for the set of $\Box$-formulas at prefix σ in branch Θ.

Definition (Symmetry Blocking)

Let $\bar{\rho}$ be a layered symmetry of φ, and Θ a branch in a tableau of φ.
Rule $(\Diamond)$ cannot be applied to $\sigma{:}\bar{\rho}(\neg\Box\psi)$ on Θ if

– it has been applied to $\sigma{:}\neg\Box\psi$
and – $\mathit{Var}(\bar{\rho}(\neg\Box\psi)) \cap \mathit{Var}(\Gamma(\sigma)) = \emptyset$,

Dynamic: $\mathit{Var}(\Gamma(\sigma))$ can change over time.

Symmetry Blocking

Completeness:

- Soundness is trivial as we do not modify the set of rules.
- Completeness requires more work.

Theorem

The tableau calculus with symmetry blocking for the modal logic $\mathcal{BML}$ is sound and complete.

Completeness lemma: a saturated open branch and its blocked $\neg\Box$-formulas form a satisfiable set of formulas.

Symmetry Blocking: Experimental Evaluation I

Implementation on the HTab solver:

- It only expands a $\neg\Box$-formula if there is no symmetric formula already expanded.
- Checks blocking condition: iff it gets a saturated open branch.
- If it holds for all blocked formulas, terminates.
- Otherwise, reschedules for further expansion.

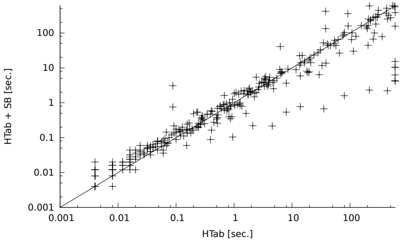
Conclusions:

- HTab+SB outperforms HTab.

Table : Total Times with SB

Solver	#Suc	#TO	T_1	T_2
HTab+SB	318	636	9657	391167
HTab	311	643	10634	396434

Figure : HTab vs. HTab+SB



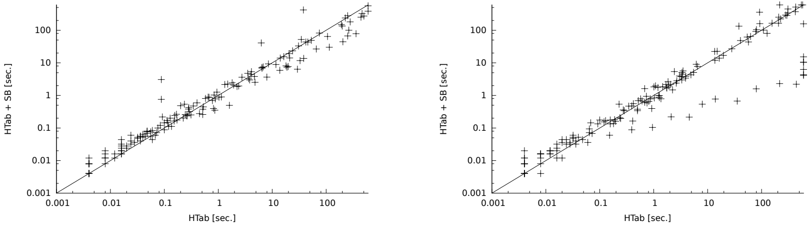
Symmetry Blocking: Experimental Evaluation II

Conclusions:

- Different behavior for sat and unsat instances.

Table : Applications of SB.

Status	#Inst	#Trig	B_1	B_2
Satisfiable	157	73	6319	6278
Unsatisfiable	163	79	1038	87



a) Satisfiable formulas b) Unsatisfiable formulas

Figure : Performance of `HTab` vs. `HTab`+SB.

Symmetry Blocking: Experimental Evaluation III

Conclusions:

- Effectiveness of SB depends on the problem class.

Table : Effect of SB on the LWB_K

Class	HTab+SB			HTab		
	n_{100}	n_{600}	T	n_{100}	n_{600}	T
k_branch_p	21	21	59.760	13	15	4402.130
k_branch_n	9	10	7010.200	8	10	7197.000
k_grz_p	21	21	0.508	21	21	0.276
k_grz_n	21	21	0.632	21	21	0.380
k_path_p	21	21	4.542	21	21	3.812
k_path_n	21	21	5.348	21	21	3.792
k_ph_p	7	8	8116.900	7	8	8095.48
k_ph_n	21	21	177.560	21	21	178.579
k_poly_p	21	21	29.068	21	21	22.949
k_poly_n	21	21	29.534	21	21	24.229

Conclusions

- Layered permutations enable to find more symmetries.
- Symmetric blocking performs differently depending on problem class.
- Not all symmetries are used by Symmetric Blocking (not always happen in $\neg - \Box$-formulas).
- Future work: Symmetry Breaking Predicates for modal logic.

Resources

- HTab prover: http://tinyurl.com/orsnu2z
- Benchmarks: http://tinyurl.com/pq63to7

Symmetries in Modal Logics: semantic properties

Property I

Let φ be a formula, ρ be a symmetry of φ and $\mathcal{M}$ a model, then, $\mathcal{M} \models \varphi$ iff $\rho(\mathcal{M}) \models \varphi$.

- Symmetries induce a partition in the model set.
- Static and Dynamic Symmetry Breaking (Symmetry Breaking Predicates).

Property II

Let φ and ψ be formulas and ρ be a symmetry of φ then, $\varphi \models \psi$ iff $\varphi \models \rho(\psi)$.

- This provides a *cheap* inference mechanism.
- Symmetric Reasoning (Symmetric Clause Learning).

This holds for permutation sequences.